Sound generation in the interaction of two isentropic vortices

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Summary: Through direct numerical simulation (DNS) for the sound generated by the interaction between two isentropic vortices, it is found that the interaction between two vortices with a large difference in their strengths or scales can generate continuous strong noise. The interaction between two isentropic vortices results in the formation of two vortex dipoles, with each vortex dipole containing two vortex cores. If there is a large difference in their initial strengths, there is a large difference in the strengths of the resulted vortex cores. As a result, the weaker vortex rotates around the stronger one continuously, which is the key source of the aerodynamic sound. This DNS result provides a new concept: the interactions among turbulent structures with large differences in their scales play a key role in the generation of turbulent noise.

Key Words: WENO scheme, direct numerical simulation, sound wave, isentropic vortex.

Since the pioneer work of Lighthill [1] in 1952 on aerodynamic sound generation, the sound generated by unsteady flows has received increasingly attention. Many theories have been developed. Typical examples include the vortex sound theory of Powell [2], the wave antenna theory of Crow [3], the instability wave model of Ffowcs Williams and Kempton [4] and the stagnation enthalpy theory of Doak [5]. These theories do offer better understandings to the generation of aerodynamic noise and provide good methods in engineering to calculate the noise generated by unsteady flows such as jet [6], wake [7], shear layer [8] and airframes [9]. However, from the earliest stress tensor theory of Lighthill [1] to the recent stagnation

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enthalpy theory of Doak [5], the theoretical sound sources are abstract and not physically measurable. How sound is produced by turbulent structures is still an open problem.

Owing to the difficulty in tackling turbulent flows, it is necessary to study a simpler flow model. Vortices are building blocks of turbulent flows. The interaction between two or among more vortices is a common phenomenon and is very important in many applications such as mixing layers, jets and combustion instability. This interaction can result in the deformation of vortices and can generate sound waves, which has a close relationship with the sound generation of turbulent flows. For example, Powell's theory of vortex sound [2] provides a framework for the turbulent sound. Under certain conditions, two co-rotating vortices can merge together. At the instant of vortex merging, strong noise is generated [10]. It has been shown that the sound generated by vortex pairing has the essential characteristics of noise generated by jets and shear layers [11, 12].

Though there are many studies [2, 10, 13] on the sound generation by vortex dominated flows, the actual mechanism is not well understood. In recent years, we have been studying the mechanism of sound generation in the interaction between a shock wave and a single planar vortex [14], a pair of planar vortices [15, 16] or a longitudinal vortex [17] through direct numerical simulation (DNS) for the two and three dimensional unsteady compressible Navier-Stokes equations using the fifth order weighted essentially nonoscillatory (WENO) finite difference scheme developed by Jiang and Shu [18] and improved by Zhang and Shu [19, 20] for the aeroacoustic computation. We have obtained many interesting findings, including the multi-stage interaction between a shock and a strong vortex [14] and limit cycles in the interaction between a normal shock wave and a longitudinal vortex [17]. The most interesting finding belongs to our recent study on the sound generated by the interaction of two isentropic vortices with a large difference in their scales, in which continuous strong noise is generated. This phenomenon might reveal the key mechanism of sound generation by turbulent flow, namely the interaction between turbulent structures with different scales, which has not been noted in previous studies. In this letter, we will briefly introduce this

finding.

As our physical model shown in Figure 1, two isentropic vortices with a separation distance d are placed in the center of our computational domain ($x_l < x < x_r$, $y_l < y < y_r$). The tangential and radial velocity, pressure and density of a single isentropic vortex [21, 14, 22] are $u_\theta(r) = Mre^{(1-r^2)/2}$, $u_r = 0$, $p(r) = \frac{1}{\gamma}[1-\frac{\gamma-1}{2}M_v^2 e^{1-r^2}]^{\frac{\gamma}{\gamma-1}}$ and $\rho(r) = [1-\frac{\gamma-1}{2}M_v^2 e^{1-r^2}]^{\frac{1}{\gamma-1}}$ respectively, where $r = \sqrt{(x-x_v)^2 + (y-y_v)^2}/r_c$, (x_v, y_v) is the center of the initial vortex, r_c is the critical radius for which the vortex has the maximum strength, M_v is the strength of the vortex, and $\gamma = 1.4$ is the ratio of specific heats. This isentropic vortex is an exact solution of the Euler equation, which can not generate any sound wave. The initial flow field for the interaction of two vortices is prescribed by the superposition of the flow fields produced by each of the two single vortices. In our simulation, the strengths of the upper and the lower vortices are chosen from 0.01, 0.2, 0.25, 0.45, 0.5, 0.75 and 0.8. The separation distance of the two vortices is set to be $d = 2{,}4$ or 2.2. The initial locations of the upper and the lower vortices are set to be $(x_u, y_u) = (0., \frac{1}{2}d)$ and $(x_d, y_d) = (0., -\frac{1}{2}d)$ respectively. r_c is set to be 1.0 or 0.2. The computational cases contain arbitrary combinations of these parameters. Our computational domain is set to be $x_l = y_l = -220$ and $x_r = y_r = 220$. After a grid convergence study to confirm enough grid resolution, we use an uniform mesh with the grid density of 6000×6000, which can offer resolved solution for the vorticity field and sound waves under study.

The interaction of two isentropic vortices has a close relationship to the rotating directions, the strengths and the initial separation distance of the two vortices. Based on the evolution of the vorticity field and the generation of sound waves in the interaction, we can classify the interaction into four modes.

Mode I: The first mode is the interaction between two counter-rotating vortices with similar scales including similar strengths and similar spacial scales. This interaction will result in a new flow structure and generate sound waves. Figure 2 is the evolution of the vorticity field in the interaction of two counter-rotating vortices with the same strength

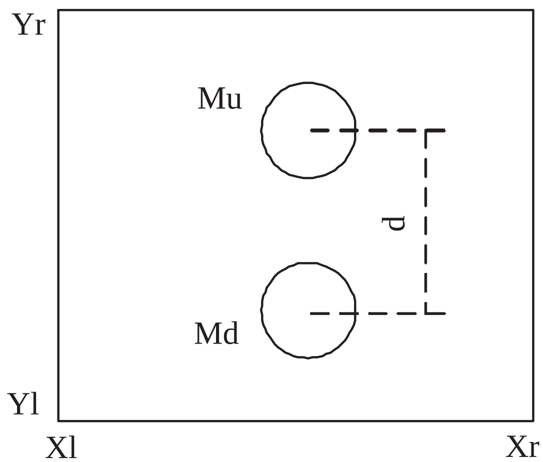
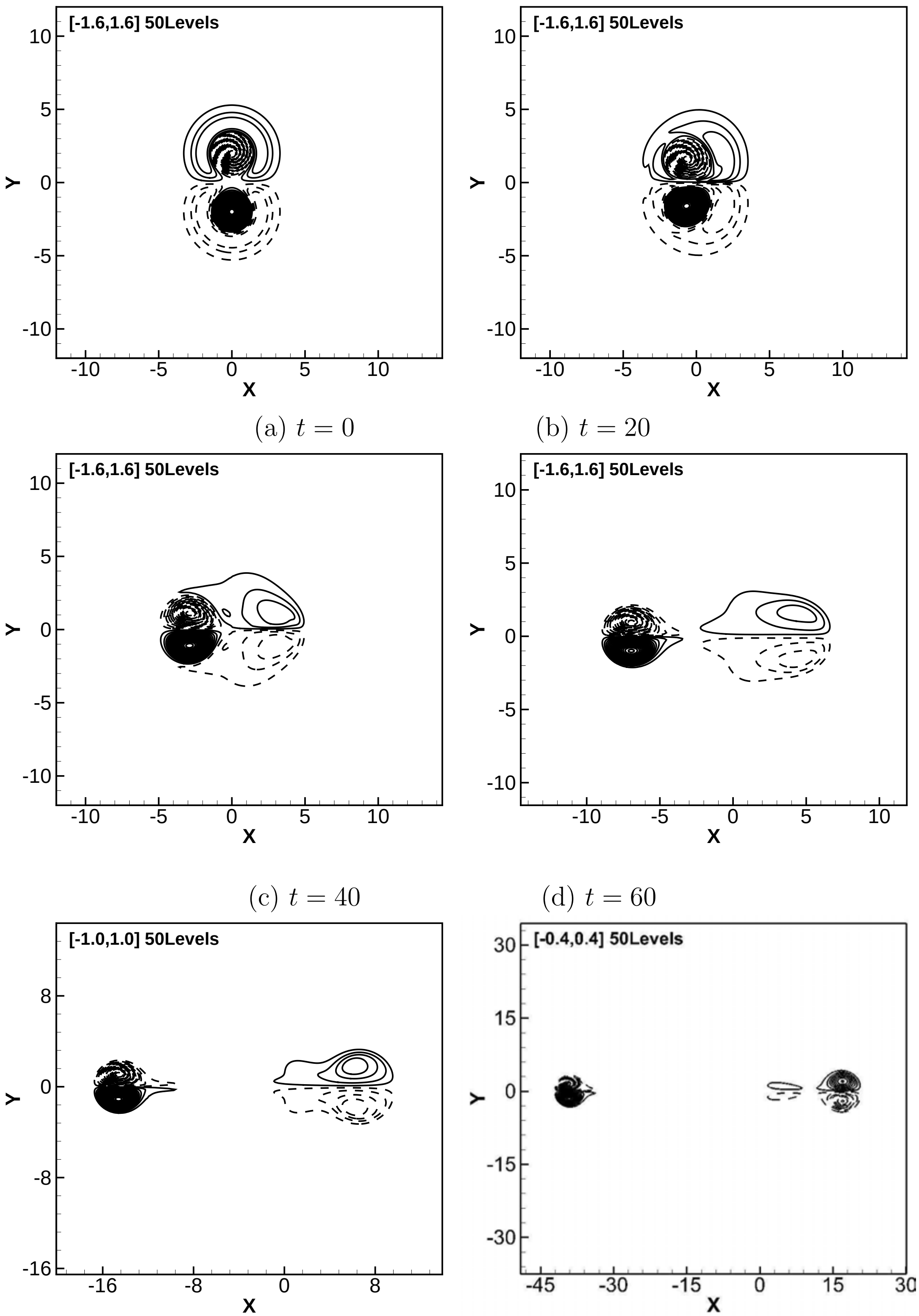


Figure 1: Schematic diagram of the flow model for the interaction of two isentropic vortices.

$M_u = -0.5$ and $M_d = 0.5$ and the same radius of $r_c = 1.0$. The upper vortex rotates clockwise and the lower vortex rotates in the opposite direction. The initial distance of the two vortices is $d = 4$. As can be seen from figure 2, the initial isentropic vortex has two layers, the inner layer and the outer layer. The sign of vorticity is opposite in the inner layer and the outer layer. The interaction results in a great change in their shapes. The vortex cores are pressed to an approximately elliptical shape from the initially circular shape and they gradually form a vortex dipole, which is advected to the left by the induced velocity. The outer layers move toward the symmetry line and gradually separate from the vortex core. They form a weaker vortex dipole which moves to the right. The vortex dipoles often appear in the wake of a school of fish [23, 24]. Five sound waves with quadrupolar nature are generated in this interaction, which are shown in figure 3 for the contours and figure 4 for the circumferential and radial (along the symmetry line) distributions of the sound pressure $\Delta p = \frac{p-p_0}{p_0}$ respectively.

Mode II: The second mode is the interaction of two co-rotating vortices in similar scales. It has the following features: (1) The interaction evolves into two non-symmetric vortex dipoles; (2) Essentially different from the interaction of two counter-rotating vortices in the first mode, two vortex cores of each vortex dipole evolve from the same initial vortex,



(a) $t = 0$ (b) $t = 20$

(c) $t = 40$ (d) $t = 60$

(e) $t = 100$ (f) $t = 300$

Figure 2: The evolution of the vorticity field in the interaction of two counter-rotating vortices in the case of $M_u = -0.5$ and $M_d = 0.5$, $d = 4$ and $r_c = 1$.

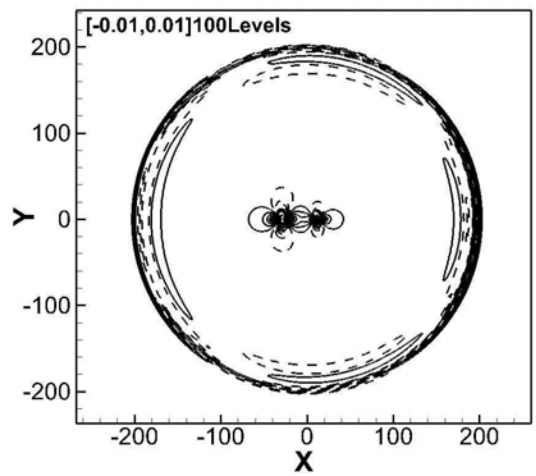


Figure 3: The contours of the sound pressure $\Delta p = \frac{p-p_0}{p_0}$ in the interaction of two counter-rotating vortices in the case of $M_u = -0.5$, $M_d = 0.5$, $d = 4$ and $r_c = 1$ at $t = 200$. Solid lines represent $\Delta p > 0$ while dashed lines represent $\Delta p < 0$.

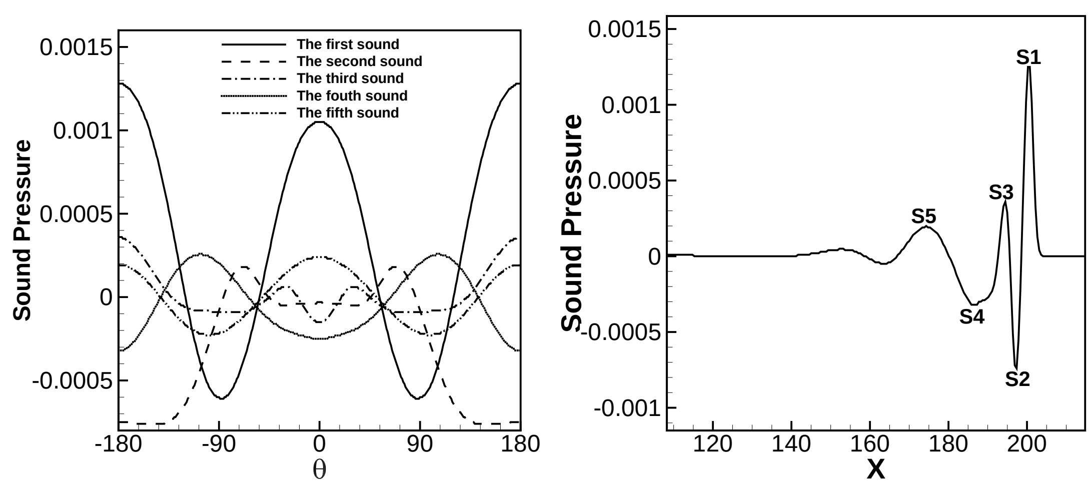


Figure 4: The circumferential (left) and radial (right) distributions of the sound pressure in the interaction of two counter-rotating vortices in the case of $M_u = -0.5$, $M_d = 0.5$, $d = 4$ and $r_c = 1$ at $t = 200$.

one resulting from the inner layer of the initial vortex and the other from the outer layer. In this process, five sound waves with quadrupolar nature are generated, which is very similar with the first Mode.

Mode III: The third mode is the merging of two co-rotating vortices. Under certain conditions, the interaction of two co-rotating vortices will result in their merger [25], which is a key phenomenon in shear layers and wake flows. Preceding the merger, the cores of the two vortices move closer, and the resulting elliptical vortices evolve into a single circular vortex with two arms. In the tail region of each arm, a weaker vortex is formed from the outer layer of the original vortex. The sign of vorticity of this weaker vortex is opposite with that of the merged core. The weaker vortices rotate in the opposite direction with the stronger ones. In the process of vortex merging, strong noise is generated which is similar to the merging process of two Gaussian vortices [10].

Mode IV: The fourth mode is the interaction of two vortices with a large difference in their scales. From the point of view of noise generation, this mode is the most interesting and is the key of this letter. This mode contains two different types. One is the interaction of two isentropic vortices with a large difference in their strengths. The other is the interaction between two isentropic vortices with a large difference in their spacial scales.

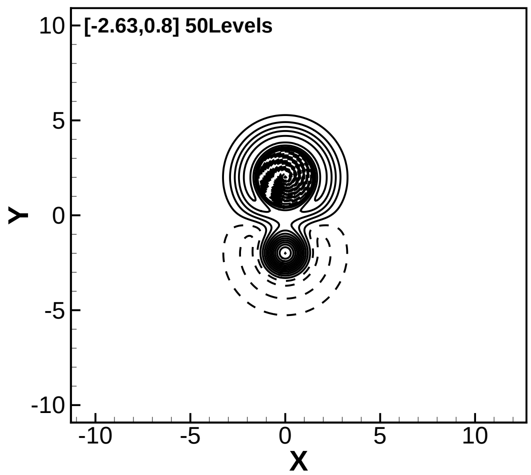
Figure 5 contains the contours of the evolution of the vorticity field in the interaction between two counter-rotating vortices with a large difference in their strengths. The strength of the upper vortex is $M_u = -0.8$ and the strength of the lower vortex is $M_d = 0.25$, which is much weaker than the upper vortex. Similar to the interaction between two counter-rotating vortices of the same strength, the interaction evolves into two vortex dipoles. The cores of the two initial vortices move closer and form a stronger vortex dipole. The outer layers separate from the initial vortices. They move closer and form a weaker vortex dipole. Because there is a large difference in their strengths, both vortex dipoles are strongly non-symmetric. As a result, the weaker vortex core rotates around the stronger one quickly. At the same time,

the weaker vortex dipole rotates around the stronger vortex dipole. In this process, strong continuous noise is generated. Figure 6 contains the contour and distribution along the symmetry line of the sound pressure at the typical time $t = 200$.

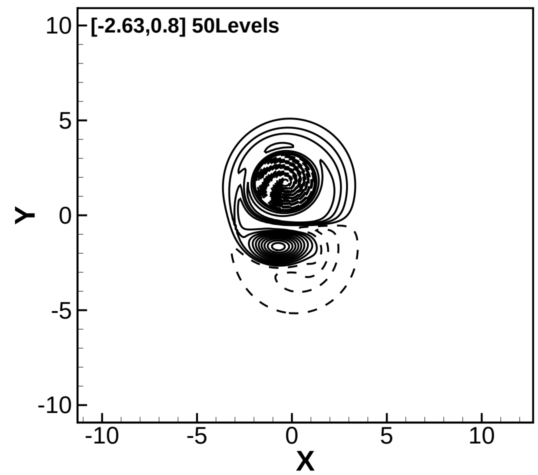
The same phenomenon is observed in the interaction of two co-rotating vortices with a large difference in their strengths. Strong continuous noise is generated, which is shown in figure 7 for the dilatation in the acoustic field at the typical time $t = 200$ for the case of $M_u = 0.8$ and $M_d = 0.25$.

To study the effect of spacial scales, we study the interaction of two vortices with large difference in their radii. Figure 8 contains the contours of the evolution of the vorticity field in the interaction between two counter-rotating vortices with a large difference in their spacial scales. The strengths of the upper and the lower vortices are the same with $M_u = -0.5$ and $M_d = 0.5$, while there is a large difference in the radii of the vortices. They are $r_c = 1.0$ and 0.2 respectively. As can be seen from figure 8, we find that the smaller vortex rotates around the larger one continuously. The initial circular vortices are pressed into elliptical shapes. This interaction can also produce continuous strong noise which is shown in figure 9.

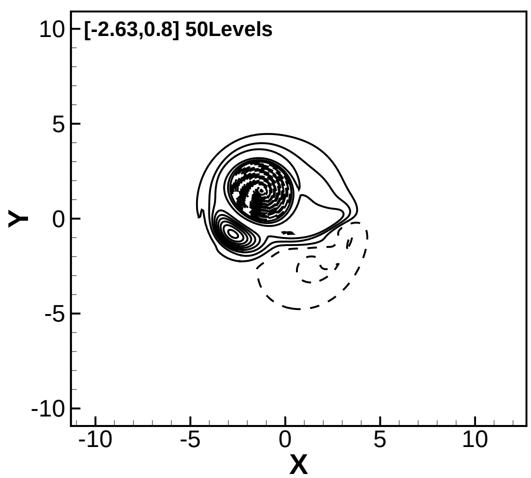
Comparing the interactions in these four modes, we find that the first mode is a basic mode. Except for the merging of two co-rotating vortices in some special conditions, two vortex dipoles are formed. If there is a large difference in their scales, either in the strengths or in spacial scales, the vortex dipoles are strongly non-symmetric. As a result, the weaker vortex core of the dipole rotates around the stronger one continuously just like a satellite. In this process, there is continuously strong noise generated. Because there are many vortices, eddies and structures of different scales in a turbulent flow, the interactions among these structures are very common. From the sound generated by the interaction of two isentropic vortices, we propose a new concept: The interactions among the structures with large differences in their scales are the key source of turbulent noise.



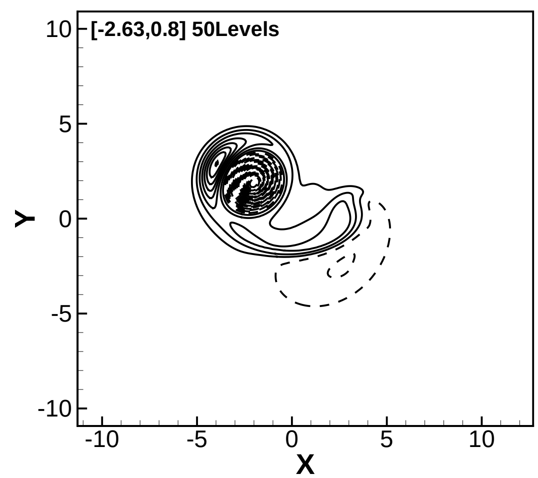
(a) $t = 0$



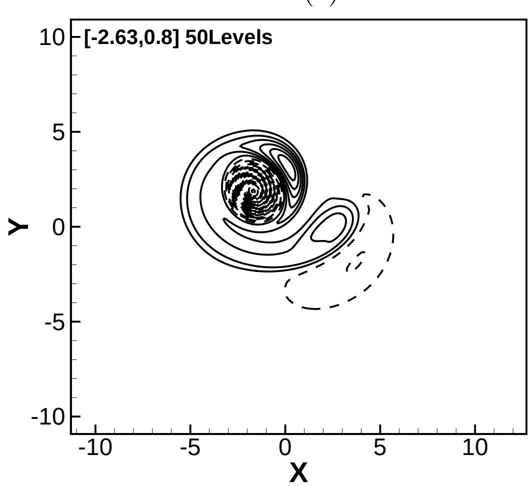
(b) $t = 20$



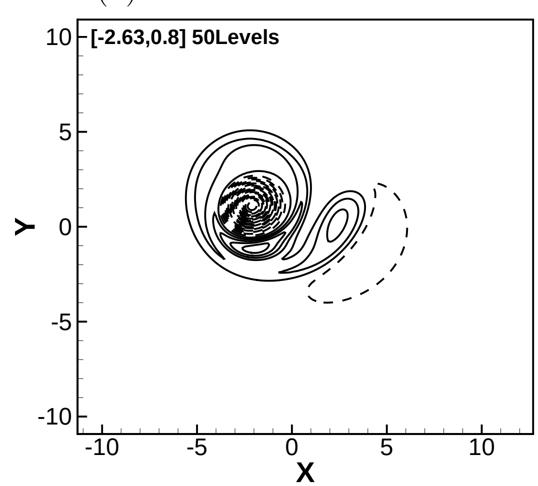
(c)$t = 40$



(d) $t = 60$



(e)$t = 80$



(f) $t = 100$

Figure 5: The evolution of the vorticity field in the interaction of two counter-rotating vortices with a large difference in their strengths. $M_u = -0.8, M_d = 0.25$, $d = 4.$ and $r_c = 1.0$.

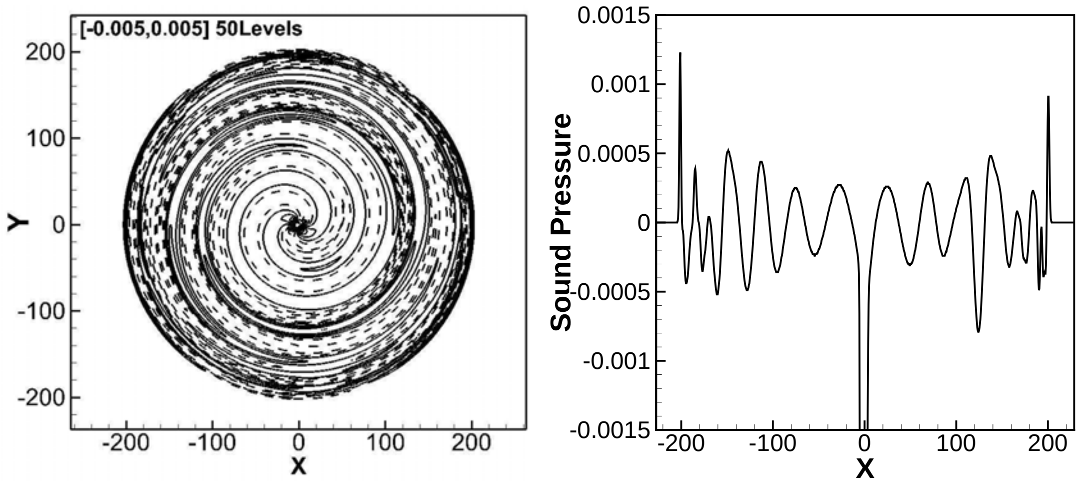


Figure 6: The contours (left) and the distribution along the symmetric line (right) of the sound pressure in the interaction of two counter-rotating vortices in the case of $M_u = -0.8$, $M_d = 0.25$, $d = 4$ and $r_c = 1$.

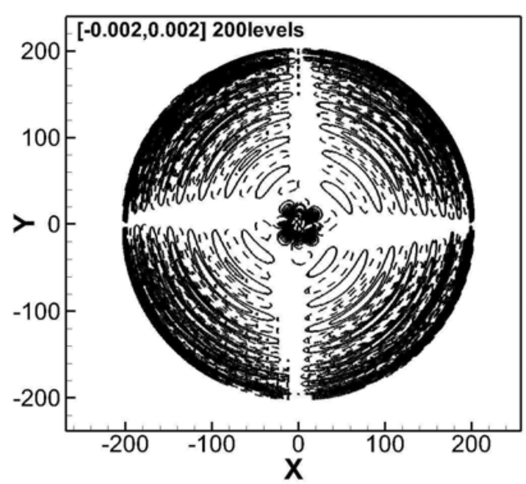
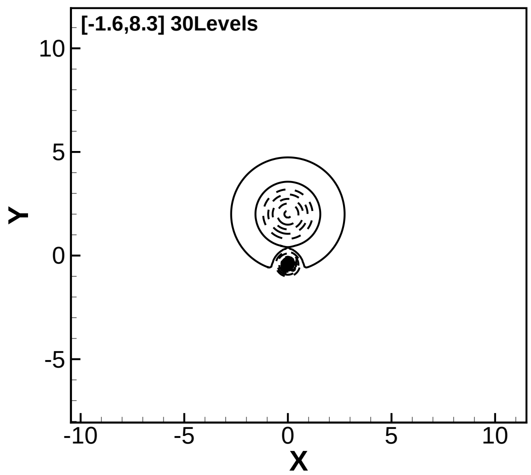
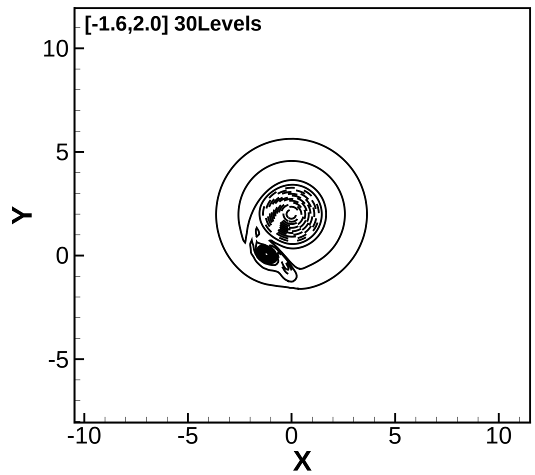


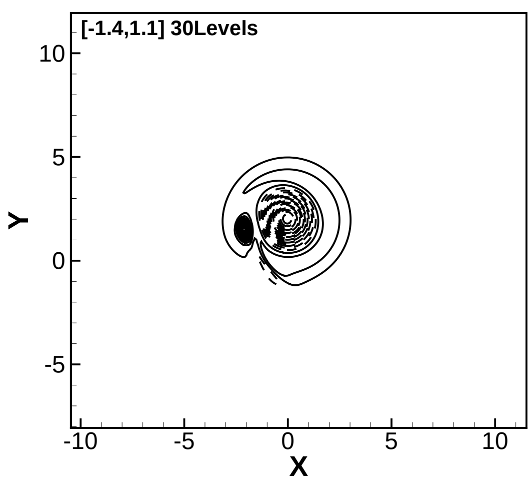
Figure 7: The contours of dilatation in the interaction of two co-rotating vortices in the case of $M_u = 0.8$, $M_d = 0.25$, $d = 4$ and $r_c = 1$ at $t = 200$.



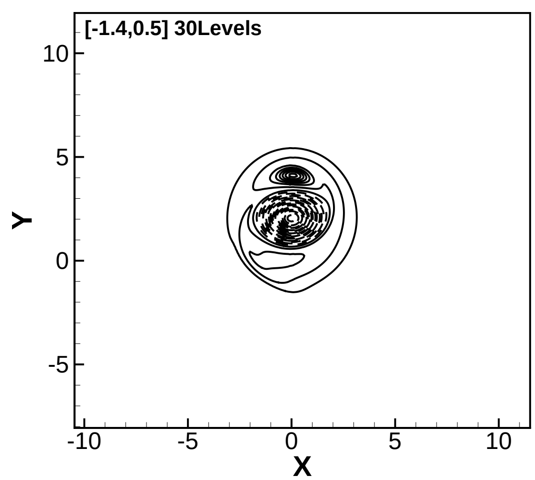
(a) $t = 0$



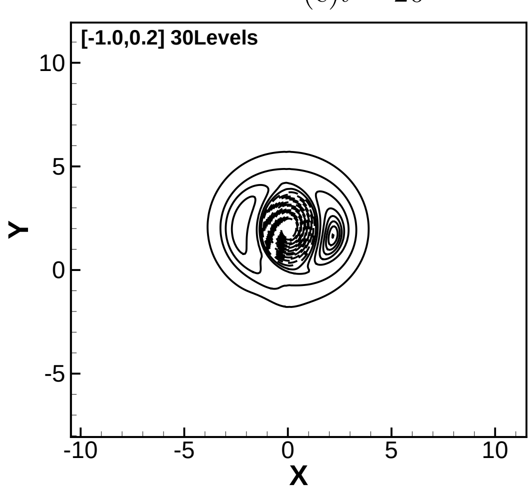
(b) $t = 10$



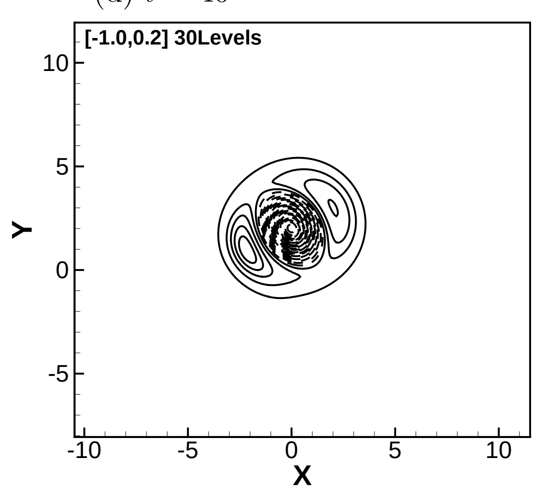
(c)$t = 20$



(d) $t = 40$



(e)$t = 60$



(f) $t = 100$

Figure 8: The evolution of the vorticity field in the interaction of two counter-rotating vortices with a large difference in their spacial scales. $M_u = -0.8$, $M_d = 0.25$, $d = 2.2$, $r_{cu} = 1$ and $r_{cd} = 0.2$.

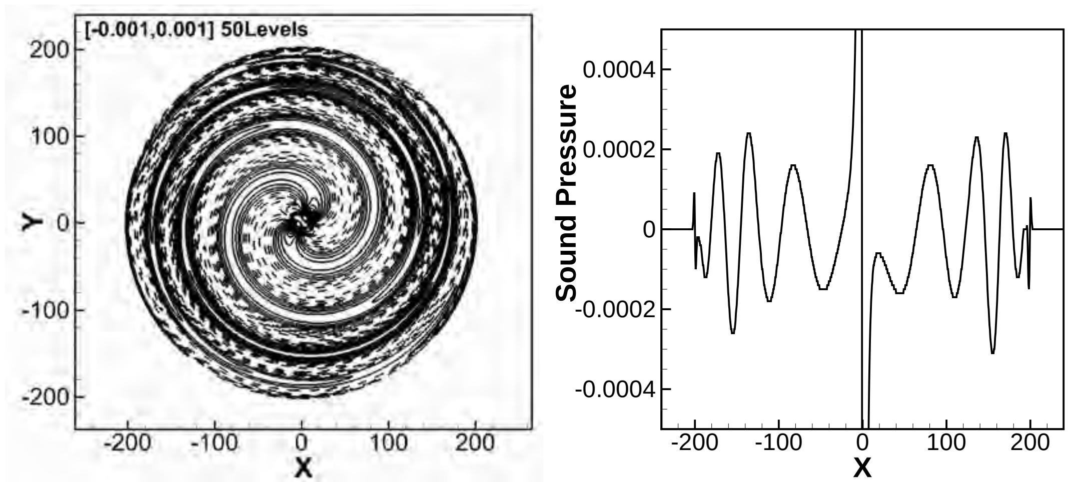


Figure 9: The contours (left) and the distribution along the symmetric line (right) of the sound pressure in the interaction of two counter-rotating vortices in the case of $M_u = -0.5$, $M_d = 0.5$, $d = 2.2$, $r_{cu} = 1$ and $r_{cd} = 0.2$.

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